Sectoral Bubbles and Endogenous Growth

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Abstract

- **Two-sector endogenous** growth model with **credit-driven stock price** bubbles.

- Bubbles’s effect
  - **Credit easing effect** (CEE): bubbles relax collateral constraints and improve investment efficiency.
  - **Capital reallocation effect** (CRE): bubbles in a sector attract more capital to be allocated to that sector.
Introduction

- **Question:** How do bubbles affect **long-run** economic growth?

- **Object:** Credit-driven bubbles
  - Accompanied by **credit expansion**.
    - Optimistic beliefs $\rightarrow$ positive feedback loop $\rightarrow$ looser credit standards and bubbles.
  
- Have **real effects** and affect market fundamentals.
  - Large elasticity of substitution $\rightarrow$ Negative capital allocation $>\$\;$ Positive capital allocation $\rightarrow$ Bubbles retard growth.

- **Appear in a particular** sector or industry.
  - Housing bubbles. internet bubble...
Introduction

- **Method**:
  - **Base paper**: Miao and Wang (2011)
  - Two-sector endogenous growth model with credit-driven stock price bubbles
    - Capital in one sector has positive externality effect on human capital.
  - Credit constraint
    - Loan repayment ≤ stock market value with the collateral.
Impact of bubbles on endogenous economic growth.

- OLS model with endogenous growth due to externality in capital accumulation.
  - Key assumption: Bubbles are on intrinsically useless assets.
  - Conclusion: Bubbles crowd out investment and reduce the growth rate of the economy.

- Infinite horizon endogenous growth AK model with financial frictions.
  - Key assumption: Bubbles are also on intrinsically useless assets, but can be used to relax collateral constraints; investment heterogeneity.
  - Conclusion: The degree of pledgeability is relatively low, bubbles enhance growth.

Key references:

- Hirano and Yanagawa (2010)
Literatures

- Bubbles in production economies with exogenous growth.
  - Introduce capital adjustment costs to the Diamond-Tirole model and study the episodes of speculative growth
    - Caballero, Farhi, and Hammour (2006)

- The effects of bubbles in the presence of financial frictions.
  - Caballero and Krishnamurthy (2006) and Farhi and Tirole (2010)
  - Conclusion: Bubbles can provide liquidity and crowd in investment.

  - Kocherlakota (2009), Martin and Ventura (2011a), and Miao and Wang (2011)
  - Conclusion: Bubbles can relax collateral constraints and improve investment efficiency.
Bubbles are attached to *productive assets*.

- Growth rate of a bubble here equals to the interest rate minus the dividend yield generated by the bubble, not the interest rate in the Tirole (1985) model.

Two *production sectors*.

- Sectoral bubbles have a capital reallocation effect between production sectors, which has not been studied in the literature.
Credit easing effect

- **Gan (2007b):** Every 10 percent drop in collateral value, the investment rate of an average firm is reduced by 0.8 percentage points.

- **Chaney, Sraer, and Thesmar (2009):** The sensitivity of investment to collateral value is stronger the more likely a firm is to be credit constrained.

- **Goyal and Yamada (2004):** Investment responds significantly to stock price bubbles. And asset price shocks primarily affect firms that rely more on bank financing and not necessarily those that use equity financing.
Credit easing effect

Figure 1: Credit easing effect using China’s data
Capital reallocation effect

Figure 2: Capital reallocation effect using China’s data
Environment

- Households, final goods producers, capital goods producers, and financial intermediaries.
- Infinite; continues-time; no aggregate uncertainty.
3.1 Household

\[
\int_{0}^{\infty} e^{-\rho t} \log(C_t) dt
\]

- Household earn labor income, trade firm stocks and make deposits to the financial intermediaries.
- Labor supply: Inelastic; normalized to 1.

\[
\frac{\dot{C}}{C} = r_t - \rho
\]  

\[r_t\]: interest rate, and equal to the rate of return on each stock.
3.2 Final Goods Producers

- Each final goods producer hires labor and rents two types of capital goods to produce output

\[ Y_t = A \omega^{\alpha-1} \left[ \omega \frac{1}{\sigma} k_{1t}^{\frac{\sigma-1}{\sigma}} + (1 - \omega) \frac{1}{\sigma} k_{2t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\alpha \sigma}{\sigma-1}} (K_{1t} N_t)^{1-\alpha} \]  

- View sector 1 as the sector producing **human capital**, sector 2 as the **manufacturing sector**.

- Knowledge has a **positive spillover effect** through human capital.

- \( k_{it} \): the stock of type \( i = 1, 2 \) capital goods rented by a final goods producer
- \( K_{it} \): the aggregate stock of type \( i \) capital
- \( A \): TFP
- \( \sigma \): the elasticity of substitution between the two types of capital
- \( \omega \): share parameter
3.2 Final Goods Producers

Final Goods Producers’ problem:

$$\max_{k_{it}, k_{2t}, N_t} A \omega^{\alpha-1} \left[ \omega \frac{1}{\sigma} k_{it}^{\frac{\sigma-1}{\sigma}} + (1 - \omega) \frac{1}{\sigma} k_{2t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\alpha\sigma}{\sigma-1}} (K_{1t} N_t)^{1-\alpha} - \omega_t N_t - R_{1t} k_{1t} - R_{2t} k_{2t}$$

F.O.C

$$\left(1 - \alpha\right) \frac{Y_t}{N_t} = w_t$$

$$A \alpha \omega^{\frac{1}{\sigma}} \omega^{\alpha-1} (K_{1t} N_t)^{1-\alpha} \left[ \omega \frac{1}{\sigma} k_{it}^{\frac{\sigma-1}{\sigma}} + (1 - \omega) \frac{1}{\sigma} k_{2t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\alpha\sigma}{\sigma-1}-1} k_{it}^{-\frac{1}{\sigma}} = R_{1t}$$

$$A \alpha (1 - \omega)^{\frac{1}{\sigma}} \omega^{\alpha-1} (K_{1t} N_t)^{1-\alpha} \left[ \omega \frac{1}{\sigma} k_{it}^{\frac{\sigma-1}{\sigma}} + (1 - \omega) \frac{1}{\sigma} k_{2t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\alpha\sigma}{\sigma-1}-1} k_{2t}^{-\frac{1}{\sigma}} = R_{2t}$$

- $w_t$: wage rate
- $R_{it}$: rental rate of type i capital
3.3 Capital Goods Producers

- **Ex ante identical** but **heterogeneous ex post** because they face idiosyncratic investment opportunities.
- This assumption captures **firm-level investment lumpiness** and generates **ex post firm heterogeneity**.
- Law of motion for capital of firm $j$ in sector $i$ between time $t$ and $t + dt$

$$K_{it+dt}^j = \begin{cases} (1 - \delta dt) K_{it}^j + I_{it}^j & \text{with probability } \pi dt \\ (1 - \delta dt) K_{it}^j & \text{with probability } 1 - \pi dt \end{cases}$$ (7)

- Capital Goods Producers’ problem

$$V_{i0} \left(K_{i0}^j\right) = \max_{I_{it}^j} \int_0^T e^{-\int_0^t r_s ds} \left(R_{it} K_{it}^j - \pi I_{it}^j\right) dt + e^{-\int_0^T r_s ds} V_{iT} \left(K_{iT}^j\right)$$ (8)
3.3 Capital Goods Producers

- Financial frictions

\[ I_{it}^j \leq R_{it}^j K_{it}^j + L_{it}^j \] (9)

- Firms do NOT raise new equity, Loans do NOT pay interest payments
  - \( L_{it}^j \): bank loans
  - \( R_{it}^j K_{it}^j \): internal funds

\[ L_{it}^j \leq V_{it} \left( \xi K_{it}^j \right) \] (10)

- Interpretation

  - **Collateral constraint**: following Kiyotaki and Moore (1997), the bank never allows the loan repayment \( L_{it}^j \) to exceed the stock market value \( V_{it} \left( \xi K_{it}^j \right) \) of the pledged assets.
  
  - **Enforcement constraint**: when the investment opportunity arrives at date \( t \); the continuation value to the firm of not defaulting is not smaller than the continuation value of defaulting.

\[ V_t \left( K_t^j + I_t^j \right) - L_t^j \geq V_t \left( K_t^j + I_t^j \right) - V_t \left( \xi K_t^j \right) \]
3.3 Capital Goods Producers

Note

- Kiyotaki and Moore (1997)’s collateral constraint: liquidation value rules out bubbles

\[ L_{it}^j \leq \xi Q_{it} K_{it}^j \]  \hspace{1cm} (11)

- \( Q_{it} \): shadow price of the capital produced in sector i.
- Here allow the collateralized assets to be valued in the stock market as the \textbf{going-concern value} when the new owner can use these assets to run the reorganized firm after default.

\[ V_{it} \left( K_{it}^j \right) = Q_{it} \left( \xi K_{it}^j \right) + B_{it} \]
3.4 Competitive Equilibrium

Let $I_{1t} = \int I_{it}^j dj$ and $K_{1t} = \int K_{it}^j dj$ denote aggregate investment and aggregate capital in sector $i$.

A competitive equilibrium consists of trajectories $(C_t), (K_{it}), (I_{it}), (Y_t), (r_t), (w_t)$, and $(R_{it}), i = 1, 2$, such that:

(i) Households optimize so that equation (1) holds.
(ii) Each firm $j$ solves problem (8) subject to (7), (9) and (10).

(ii) Rental rates satisfy:

$$R_{1t} = A\alpha\omega^{\frac{1}{\sigma}}\omega^{\alpha - 1}(K_{1t}N_t)^{1-\alpha}\left[\omega^{\frac{1}{\sigma}} k_{it}^{\frac{\sigma-1}{\sigma}} + (1 - \omega)^{\frac{1}{\sigma}} k_{2t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\alpha\sigma}{\sigma-1} - 1} k_{it}^{-\frac{1}{\sigma}}$$

(12)

$$R_{2t} = A\alpha(1 - \omega)^{\frac{1}{\sigma}}\omega^{\alpha - 1}(K_{1t}N_t)^{1-\alpha}\left[\omega^{\frac{1}{\sigma}} k_{it}^{\frac{\sigma-1}{\sigma}} + (1 - \omega)^{\frac{1}{\sigma}} k_{2t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\alpha\sigma}{\sigma-1} - 1} k_{2t}^{-\frac{1}{\sigma}}$$

(13)
3.4 Competitive Equilibrium

(iii) The wage rate satisfies (4) for $N_t = 1$.

(iv) Markets clear in that:

$$C_t + \pi (I_{1t} + I_{2t}) = Y_t = A\omega^{\alpha-1} \left[ \omega \frac{1}{\sigma} k_{1t}^{\frac{\sigma - 1}{\sigma}} + (1 - \omega) \frac{1}{\sigma} k_{2t}^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\alpha \sigma}{\sigma - 1}} (K_{1t})^{1-\alpha}$$

(14)

To write equations (12), (13), and (14), we have imposed the market-clearing conditions $k_{it} = K_{it}$ and $N_t = 1$ in equations (5), (6), and (2).
4.1 A Single Firm’s Decision Problem

**Proposition 1:** Suppose $Q_{it} > 1$, then:

(i) The market value of the firm is given by (15);

$$V_{it} \left( K_{it}^j \right) = Q_{it} K_{it}^j + B_{it}$$  \hspace{1cm} (15)

If $B_{it} > 0$, then

$$L_{it}^j \leq V_{it} \left( \xi K_{it}^j \right) = \xi Q_{it} K_{it}^j + B_{it}$$  \hspace{1cm} (16)

(ii) Optimal investment is given by:

$$I_{it}^j = (R_{it} + \xi Q_{it}) K_{it}^j + B_{it}$$  \hspace{1cm} (17)

(iii) $(B_{it}; Q_{it})$ satisfy the following differential equations:

$$r_t Q_{it} = R_{it} + (R_{it} + \xi Q_{it}) \pi (Q_{it} - 1) - \delta Q_{it} + \ddot{Q}_{it}$$  \hspace{1cm} (18)

$$r_t B_{it} = \pi (Q_{it} - 1) B_{it} + \dot{B}_{it}$$  \hspace{1cm} (19)
4.1 A Single Firm’s Decision Problem

(vi) And the transversality condition:

\[
\lim_{T \to \infty} \exp \left( - \int_0^T r_s ds \right) Q_i T K_{iT}^j = 0, \quad \lim_{T \to \infty} \exp \left( - \int_0^T r_s ds \right) Q_i T B_{iT}^j = 0
\] (20)

- \(Q_{it}\): determined variable; the shadow value of capital.
- \(B_{it}\): determined variable: bubble component.

\[
\frac{\dot{B}_{it}}{B_{it}} + \pi (Q_{it} - 1) = r_t \quad \text{for } B_t > 0
\] (21)

- Bubbles are on productive assets and their growth rate is less than the interest rate, therefore they cannot be ruled out by the transversality condition.
4.2 Equilibrium System

**Proposition 2**: Suppose $Q_{it} \not= 1$, then the equilibrium dynamics for $(B_{it}, Q_{it}, K_{it}, I_{it}, C_t, Y_t)$ satisfy the following system of differential equations:

\[
\begin{align*}
\dot{K}_{it} &= -\delta K_{it} + \pi I_{it}, \quad K_{i0} \text{ given} \quad \text{(22)} \\
I_{it} &= (R_{it} + \xi Q_{it}) K_{it} + B_{it} \quad \text{(23)}
\end{align*}
\]

Together with (14), (18)-(19), and the transversality condition:

\[
\lim_{T \to \infty} \exp \left( - \int_0^T r_s ds \right) Q_{iT} K_{iT} = 0, \quad \lim_{T \to \infty} \exp \left( - \int_0^T r_s ds \right) Q_{iT} B_{iT} = 0 \quad \text{(24)}
\]

where $R_{1t}$ and $R_{2t}$ satisfy (12) and (13), respectively, and $r_t$ satisfies (1).
4.3 Bubbleless Equilibrium $B_{it} = 0$

- **Without** financial frictions: $Q_{it} = 1$

\[
R_{1t} = R_{2t} = R^* \equiv \alpha A
\]  
(25)

\[
\frac{\omega}{1 - \omega} = \frac{K_{1t}}{K_{2t}}
\]  
(26)

\[K_{1t} = \omega K_t K_{2t} = (1 - \omega) K_t \text{ if define } K_t = K_{1t} + K_{2t}
\]  
(27)

\[Y_t = A \omega^{\alpha - 1} K_{1t}^{1 - \alpha} \left[ \omega \frac{1}{\sigma} k_{it}^{\frac{\sigma - 1}{\sigma}} + (1 - \omega) \frac{1}{\sigma} k_{2t}^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\alpha \sigma}{\sigma - 1}} = AK_t
\]  
(28)

\[g_0 = \alpha A - \rho - \delta < A - \rho - \delta \text{ (first-best).}
\]

- **With** financial frictions: $Q_{it} > 1$, collateral constraints are binding.

\[g^* = \frac{K_{it}}{K_{it}} = -\delta + \pi (R_{it} + \xi Q_{it})
\]  
(29)

\[(r + \delta) Q^* = R^* + \pi (R^* + \xi Q^*) (Q^* - 1)
\]  
(30)

\[Q^* = \frac{(1 - \pi) \alpha A}{\rho + \pi \xi}
\]  
(31)
4.3 Bubbleless Equilibrium  $B_{it} = 0$

Proposition 3: Suppose

$$\alpha A - \rho - \delta > 0 \quad (32)$$

(i) If

$$\xi > \frac{\alpha A (1 - \pi)}{\pi} - \frac{\rho}{\pi} \quad (33)$$

then consumption, capital, and output on the balanced growth path grow at the rate

$$g_0 = \alpha A - \rho - \delta \quad (34)$$

(ii) If

$$\xi < \frac{\alpha A (1 - \pi)}{\pi} - \frac{\rho}{\pi} \quad (35)$$

$$\frac{\alpha A \pi (\rho + \xi)}{\rho + \pi \xi} > \delta \quad (36)$$

then the balanced growth path grow at the rate

$$g^* = \frac{\alpha A \pi (\rho + \xi)}{\rho + \pi \xi} - \delta < g_0 \quad (37)$$
4.3 Bubbleless Equilibrium \( B_{it} = 0 \)

To ensure a balanced growth path when collateral constraints bind, \( \xi \) should satisfy:

\[
\frac{\rho (\delta - \alpha A \pi)}{\pi (\alpha A - \delta)} < \xi < \frac{\alpha A (1 - \pi)}{\pi} - \frac{\rho}{\pi}
\]

**Intuition** behind the determinant of growth:

\[
g = -\delta + \frac{\pi (I_{1t} + I_{2t})}{K_{1t} + K_{2t}} = -\delta + s \frac{Y_t}{K_t}
\] (38)

where the aggregate investment rate or the aggregate saving rate

\[
s = \frac{\pi (I_{1t} + I_{2t})}{Y_t} = \frac{\alpha \pi (\rho + \xi)}{\rho + \pi \xi}
\]

- Both \( s \) and the \( Y/K \) are **constant** along a balanced growth path.
- Larger \( \pi \) \( \Rightarrow \) more investment opportunities \( \Rightarrow \) larger \( g^* \)
- larger \( \xi \) \( \Rightarrow \) looser collateral constraints \( \Rightarrow \) raising the investment rate.
5. Symmetric Bubbly Equilibrium, $B_{it} > 0$

On the balanced growth path, the growth rate $g_b$ satisfies:

$$ r = g_b + \rho $$

(39)

Because

$$ r = g_b + \pi (Q_{it} - 1) $$

(40)

Therefore, the capital price on the balanced growth path $Q_b$

$$ Q_{1t} = Q_{2t} = Q_b = \frac{r - g_b}{\pi} + 1 = \frac{\rho}{\pi} + 1 $$

(41)

Therefore, the interest rate $R_b$ satisfies:

$$ (r + \delta)Q_b = R_{it} + \pi (R_{it} + \xi Q_b)(Q_b - 1) $$

(42)

Because

$$ R_{1t} = R_{2t} = R_b = \alpha A $$

(43)

- Two sectors face the same degree of financial frictions $\implies$ the presence of bubbles in both sectors does not distort capital allocation across the two sectors.
5. Symmetric Bubbly Equilibrium, $B_{it} > 0$

**Proposition 4:** Suppose condition (36) and the following condition hold:

$$\xi < \frac{\alpha A (1 - \pi)}{\rho + \pi} - \frac{\rho}{\pi}$$  \hspace{1cm} (44)

Then, on the balanced growth path,

(i) Both the bubbleless equilibrium and the symmetric bubbly equilibrium exist;

(ii) The economic growth rate in the symmetric bubbly equilibrium is given by:

$$g_b = \frac{(1 + \rho) \alpha A}{\rho + \pi} + \rho \xi - \rho - \delta$$  \hspace{1cm} (45)

(iii) $g^* < g_b < g_0$ Because

$$g_b = \frac{\dot{K}_{it}}{K_{it}} = -\delta + \pi \left( R_b + \xi Q_b + \frac{B_{it}}{K_{it}} \right)$$  \hspace{1cm} (46)
6.1 Bubbles in the Sector with Externality

If $B_{1t} > 0$, and $B_{2t} = 0$, then on the balanced growth path

$$r = g_{1b} + \rho$$  \hspace{1cm} (47)

$$r = g_{1b} + \pi (Q_1 - 1)$$  \hspace{1cm} (48)

Therefore

$$Q_1 = \frac{\rho}{\pi} + 1 > 1$$  \hspace{1cm} (49)

Because

$$(r + \delta) Q_1 = R_{1t} + \pi (R_{1t} + \xi Q_1) (Q_1 - 1)$$  \hspace{1cm} (50)

Therefore

$$R_1 = \frac{1}{\pi} \frac{\rho + \pi}{1 + \rho} [\rho (1 - \xi) + \delta + g_{1b}]$$  \hspace{1cm} (51)

From the F.O.C of final goods producers,

$$\left( \frac{R_{1t}}{R_{2t}} \right)^\sigma = \frac{\omega}{1 - \omega} \frac{K_{2t}}{K_{1t}}$$  \hspace{1cm} (52)
6.1 Bubbles in the Sector with Externality

Then

\[
R_1 = A\alpha \omega^{\frac{1}{\sigma}} + \alpha - 1 \left[ \omega^{\frac{1}{\sigma}} + \frac{1 - \omega}{\omega^{\frac{1}{\sigma}}} \left( \frac{R_1}{R_2t} \right)^{\sigma - 1} \right]^{\frac{\alpha\sigma - \sigma + 1}{\sigma - 1}}
\]  

(53)

\(R_{1t} = R_1\) and \(R_{2t} = R_2\) are constant. Because

\[g_{1b} = \frac{\dot{K}_{1t}}{K_{1t}} = -\delta + \pi \left( R_1 + \xi Q_1 + \frac{B_{1t}}{K_{1t}} \right)\]  

(54)

\[g_{1b} = \frac{\dot{K}_{2t}}{K_{2t}} = -\delta + \pi \left( R_2 + \xi Q_{2t} \right)\]  

(55)

Thus, \(Q_{2t} = Q_2\) is also equal to a constant. Since

\[\left( \rho + g_{1b} + \delta \right) \underbrace{Q_2}_{r} = R_2 + \pi \left( R_2 + \xi Q_2 \right) \left( Q_2 - 1 \right)\]  

(56)
6.1 Bubbles in the Sector with Externality

Therefore,

\[ Q_2 = \frac{1 - \pi}{\pi \xi + \rho} R_2 \]  

(57)

Substituting this equation into (55) yields

\[ R_2 = \frac{1}{\pi} \frac{\pi \xi + \rho}{\xi + \rho} (\delta + g_{1b}) \]  

(58)

(51), (58) and (53) yields a nonlinear equation for \( g_{1b} \).

**Proposition 5:** Suppose that there exists a unique solution \((R_1, R_2, g_{1b})\) to the system of equations (51), (53) and (58). Suppose that:

\[ g_{1b} > \frac{(\xi + \rho)(\pi + \rho)}{1 - \pi} - \delta > 0 \]  

(59)

Then the steady-state asymmetric bubbly equilibrium with \( B_{1t} > 0 \) and \( B_{2t} = 0 \) exists and the economic growth rate is \( g_{1b} \).

**Note**

\( B_{1t}/K_{1t} > 0, Q_2 > 1 \) and \( Q_1 > 1 \) \( \implies \) \( R_1 < R_2 \)
6.2 Bubbles in the Sector without Externality

If $B_{1t} = 0$, and $B_{2t} < 0$, then on the balanced growth path,

Proposition 6 Suppose that there exists a unique solution $(R_1, R_2, g_{2b})$ to the following system of equations:

$$R_1 = \frac{1}{\pi} \frac{\pi \xi + \rho}{\rho + \xi} (\delta + g_{2b})$$

$$R_2 = \frac{1}{\pi} \frac{\pi + \rho}{\rho + 1} \left[ \rho (1 - \xi) + \delta + g_{2b} \right]$$

(60)

(61)

together with (53). Suppose that

$$g_{2b} > \frac{(\xi + \rho)(\pi + \rho)}{1 - \pi} - \delta > 0$$

(62)

Then the steady-state asymmetric bubbly equilibrium with $B_{1t} = 0$, and $B_{2t} < 0$ exists and the economic growth rate is $g_{2b}$.

Note

$B_{2t}/K_{2t} > 0, Q_2 > 1$ and $Q_1 > 1 \implies R_2 < R_1$
6.3 Do Bubbles Enhance or Retard Growth?

Proposition 7 Suppose that the conditions in Propositions 3(ii) and 4-6 hold.

(i) If $\sigma > 1/(1 - \alpha)$, then

$$ g_{2b} < g^* < g_b < g_{1b} $$

(ii) If $0 < \sigma < 1/(1 - \alpha)$, then

$$ g^* < g_{1b} < g_b g^* < g_{2b} < g_b $$

(iii) If $\sigma = 1/(1 - \alpha)$, then

$$ g_{2b} = g^* < g_b = g_{1b} $$
6.3 Do Bubbles Enhance or Retard Growth?

Define $\beta = K_{1t}/K_t$, then

$$\frac{Y_t}{K_t} = A\omega^{\alpha-1} \left[ \omega \frac{1}{\sigma} \beta^{\frac{\sigma-1}{\sigma}} + (1 - \omega) \frac{1}{\sigma} (1 - \beta)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\alpha\sigma}{\sigma-1}} \beta^{1-\alpha} \quad (63)$$

$$g = -\delta + \frac{\pi (I_{1t} + I_{2t})}{K_{1t} + K_{2t}} = -\delta + s \frac{Y_t}{K_t} \quad (38)$$

- Credit easing effect (CEE) relaxes the collateral constraints and raises the aggregate saving rate $s$
- Capital reallocation effect (CRE) influences capital allocation between the two sectors represented by $\beta$
6.3 Do Bubbles Enhance or Retard Growth?

- Bubbleless and the symmetric bubbly equilibria, $\beta = \omega$ but $s$ goes up, therefore no CRE but only CEE, $\implies g_b > g^*$. 

- Asymmetric bubbles, $\beta > \omega$.
  - $B_{1t} > 0$,
    then the externality effect raises the $Y/K$. CRE let $g_{1b}$ go up. But since CEE decrease, then $g_{1b}$ goes down. Only when $\sigma > \frac{1}{1-\alpha}$, CRE dominates, the $g_b < g_{1b}$.

  - $B_{2t} > 0$,
    - CRE is negative compare to symmetric bubbly equilibria due to the less productive sector 2. And CEE decrease, therefore $g_{2b} < g_b$.
    - CRE is negative but CEE is positive compare to bubbleless equilibria due to the bubble in sector 2. Therefore only $\sigma > \frac{1}{1-\alpha}$, CRE’s effect dominates, $g_{2b} > g^*$. 
This paper showed...

- Bubbles have a **credit easing effect** in that they relax collateral constraints and improve investment efficiency.

- Sectoral bubbles also have a **capital reallocation effect** in that bubbles in one sector attracts capital to be reallocated to that sector.

- If the **elasticity of substitution** between the two types of capital goods is relatively large, then the capital reallocation effect will **dominate** the credit easing effect.

- In this case, the existence of bubbles in the sector that does not generate externality will **reduce** long-run growth. Bubbles may occur in the other sector that generates positive externality or in both sectors. In these cases, the existence of bubbles **enhances** economic growth.